# 目　　錄

# 一、學術獎

## 謝銘倫 研究員

### 學歷 Education：

美國哥倫比亞大學博士, 2008
國立台灣大學數學系碩士,2000
國立台灣大學數學系學士,1998

### 經歷 Experience：

中央研究院數學研究所研究員 2016-
台灣大學數學系副教授 2013－2016
台灣大學數學系助理教授2009－2013

### 研究領域 Research interests：

數論

### 曾獲得之學術獎勵

| | |
|---|---|
| 科技部傑出研究獎 | 2014 |
| 中央研究院年輕理論學者獎 | 2014 |
| 吳大猷先生紀念獎 | 2013 |
| TIMS 第二屆傅斯年數學獎 | 2013 |
| 國家科學理論中心年輕理論學者獎 | 2013 |
| 中華民國數學會青年數學家獎 | 2012 |

### 推薦理由

Dr. Ming-Lun Hsieh is an expert in algebraic number theory, especially in Iwasawa theory and special values of L-functions. His outstanding contributions in understanding Iwasawa main conjecture for CM fields and the p-adic BSD conjecture for CM elliptic curves are highly regarded among the top world experts. In less than ten years Dr. Hsieh has establish himself as one of the top world expert in algebraic number theory and is generally considered one of the main authorities in Iwasawa theory. Dr. Hsieh's recent works concerning the construction of p-adic tripe L-series associated to the so-called Hida families are of the highest standards and they are expected to have many further important applications in the future. Besides making outstanding scientific contributions, Dr. Hsieh has supervised many Ph.D. students and post-doctoral fellows since coming back to Taiwan. This is despite his relatively young

age and shows that he is very much concerned about nurturing the next generation of mathematicians in Taiwan. It is our expectation that he will continue to play a very significant role in this regards in the years to come. In conclusion, I believe that he is currently the most natural candidate for the Academic Award of the Mathematica Society and recommend him in the strongest possible term.

兩位專家學者對謝銘倫教授研究成果給予如下之綜合評語：

1. Dr. Ming-Lun Hsieh's research area is algebraic number theory. He has made signification contributions to the Iwasawa main conjecture for CM field and the p-adic BSD conjecture for CM elliptic curves. His work appeared in JAMS in 2014 is highly recognized by the word experts in this area. His more recent work on the arithmetic of p-adic triple L-series also makes important contributions to modular forms and elliptic curves.

   Dr. Hsieh has an impressive publication list with 17 papers and preprint in 10 years after receiving the PhD degree. He has established himself as one of the leading expert in the Iwasawa theory.

2. Prof. Hsieh is an internationally well-known expert in the area of Birch and Swinnerton-Dyer (BSD) conjecture. This is a notoriously difficult, deep and important area in number theory, to which Hsieh is steadily making groundbreaking contributions. Working in this area requires mastering automorphic forms/representations of unitary and symplectic groups, special values of p-adic and complex L-functions, Galois cohomology, Euler systems, etc. In his effort to establish the BSD conjecture for various cases, Hsieh has made contributions to each of the above topics. His papers all appear in top ranked journals, including Journal of AMS, which speak volume of the importance and quality of his work.

   Prof. Hsieh has received many awards in the past. He is a leading number theorist in Taiwan who enjoys high international reputation and visibility. People flock to him to seek his supervision and guidance, including some from US and Japan. This is a rare phenomenon among Taiwanese mathematicians. I admire his achievements greatly. He definitely deserves the honor and recognition of this prestigious award from Taiwanese Math. Society. I recommend him strongly for this award.

綜觀兩位專家學者對謝教授研究所給高度肯定之評語，本會特頒學術獎，以茲表揚。

## 研究工作介紹

**1. Iwasawa main conjecture for CM fields (2010-2013)**

The work [1] is the culmination of my three-year effort on Iwasawa theory for CM fields, in which I proved an one-sided divisibility result towards Iwasawa main conjecture for CM fields as well as p-adic Birch and Swinnerton-Dyer conjecture for CM elliptic curves over totally real fields. The method is to construct Eisenstein congruence on the quasi-split unitary group of degree three and then apply a generalization of Ribet' s converse of Herbrand to construct non-trivial Selmer classes for CM fields. The hardest part was to prove the Eisenstein congruence I construct is non-trivial. To resolve this difficulty, I was led to prove the vanishing of mu-invariants of anticyclotomic p-adic L-fucntions for CM fields in [2], where for the first time the theta dichotomy is applied in Iwasawa theory.

**2. Anticyclotomic Iwasawa theory for modular forms (2014-2016)**

With Masataka Chida, I developed Iwasawa theory for elliptic modular forms of higher weights over the anticyclotomic Zp-extension of an imaginary quadratic field. The analytic side was carried out in our Crelle paper (2018). In the algebraic side, we proved one-sided anticyclotomic Iwasawa main conjecture for modular forms in our Compositio paper (2015). With Francesc Castella, I proved Perrin-Riou' s explicit reciprocity law for generalized Heegner cycles over the anticyclotomic Zp-extension of an imaginary quadratic field, and established Bloch-Kato conjecture for modular forms over imaginary quadratic fields in the rank zero case in our Math. Ann. Paper (2018).

**3. Arithmetic of p-adic triple L-series (2017--present)**

In the work [3], I constructed the three variable p-adic triple L-series associated with Hida families. As an application, I gave an Euler system construction of the anticyclotomic p-adic L-functions for modular forms. This leads to my current joint work with Francesc Castella, where we prove the non-vanishing of generalized Kato class for an elliptic curve E is equivalent to the p-Selmer rank of E is two. This work gives the first explicit construction of a non-trivial Selmer class for elliptic curves of rank two and deserves of further investigation.

## 代表著作

1. Eisenstein congruence on unitary groups and Iwasawa main conjecture for CM fields Journal of the American Mathematical Society, 27(2014), no.3, 753-862.
2. On the vanishing of mu-invariant of anticyclotomic p-adic L-functions for CM fields Journal fur die Reine und Angewandte Mathematik, 688(2014), 67-100.
3. Hida families and p-adic triple product L-functions American Journal of Mathematics (2019), to appear. 78pp.

# 二、青年數學家獎

## 魏福村 副教授

### 學歷 Education：

國立清華大學數學所博士, 2005-2010
國立清華大學數學所碩士, 2003-2005
國立台灣師範大學數學系學士,1999-2003

### 經歷 Experience：

國立清華大學副教授, 2018-
國立中央大學助理教授, 2016-2018
中央研究院數學所研究學者, 2013-2016
國立清華大學博士後研究員, 2010-2013
(研發替代役)

### 研究領域 Research interests：

數論 (函數體上的算術和L函數)

### 曾獲得之學術獎勵

| | |
|---|---|
| 科技部傑出研究獎 | 2019 |
| 科技部吳大猷先生紀念獎 | 2018 |
| 傑出人才發展基金會第六屆年輕學者創新獎 | 2018 |
| 國家理論科學研究中心年輕理論學者獎 | 2016 |
| 科技部博士後研究人員學術著作獎 | 2012 |
| 中華民國數學會傑出碩博士論文獎（金牌） | 2010 |

### 推薦理由

國內兩位專家學者對魏福村副教授研究成果給予如下之綜合評語：

1. Prof. Fu-Tsun Wei's research area is number theory. More specifically, he is working on arithmetic of curves over function fields, special values of L-functions over function fields and arithmetic of Drinfeld modules. He is known for his work of using automorphic forms over function fields to study the special values of L-functions on the ellipticcurves.

   Prof. Wei obtained his PhD in 2010. In less than 10 years, Dr. Wei has an impressive list of publications with 19 papers and preprints. He is considered as a promising young star in the area of number theory.
2. Prof. Wei's research focuses on function field arithmetic. More precisely, he studies arithmetic properties of special values of L-functions attached to algebraic

objects, like curves, varieties, and Drinfeld modules, defined over function fields, finds their geometric meaning as well as connections with automorphic functions. He has published many important papers in number theory and arithmetic geometry in the very top notch journals. It is very impressive in terms of both quality and quantity.

Prof. Wei undoubtedly is the most outstanding young mathematician in Taiwan of his generation, as evidenced by the many awards he has won in his career.

While his performance has always been at the top among his peers in Taiwan ever since his Ph.D., it should be pointed out that today he has built an outstanding international reputation. In view of the increasing slope of his performance, it is very assuring to me that more remarkable results can be expected of him in the future. He absolutely deserves the honor and recognition of this award from the Taiwanese Math. Society.

綜觀兩位專家學者對魏副教授研究所給高度肯定之評語，本會特頒青年數學家獎，以茲表揚。

## 研究工作介紹

The research of the nominee focuses on special values of the automorphic L-functions and Eisenstein series over function fields. In the function field world, the “Drinfeld modular parametrization” always exists for every non-isotrivial elliptic curve. In particular, the Hasse-Weil L-function of non-isotrivial elliptic curves are “automophic”. Moreover, from the work of Tate, we have the “one-side inequality” in the conjecture of Birch and Swinnerton-Dyer over function fields. The work of the nominee is mainly to use automorphic forms over function fields as tools to obtain explicit formulas for special values of the L-functions coming from non-isotrivial elliptic curves.

One of his current work is to derive a function field analogue of the Kronecker limit formula, which expresses the logarithmic derivative of “non-holomorphic” Eisenstein series on “Drinfeld half space” in terms of the “degree” of the corresponding “period discriminant”. As a consequence, this formula enables us to connect the logarithmic derivative of the Dedekind-Weil zeta function associated to the “imaginary” (as well as the“totally real”) function fields with an average of the “co-volume” of the corresponding “period lattices”. This leads directly to a Colmez-type formula for “CM Drinfeld modules”. Note that Eisenstein series can be viewed as “kernel functions” in the integral forms of various types of automorphic L-functions. His formula also gives a “period” interpretation for the associated L-values.

Another work of the nominee introduces non-archimedean analogue of

“automorphic” Green’s functions, and shows a “limit formula” which expresses their derivatives in terms of the Manin-Drinfeld theta functions on Drinfeld half plane. From Gross’ interpretation of Neron’s local height in the non-archimedean case, the derivative in question are related naturally to certain “intersection numbers”. One key observation (by the nominee) on the connection between Eisenstein series and automorphic Green’s functions over function fields, this result provides a conceptual link between the corresponding automorphic L-values and certain “geometric quantities”.

## 代表著作

1. Fu-Tsun Wei, Kronecker limit formula over global funct ion f ields, American Journal of Mathemat ics vol. 139 no. 4 (2017)1047-1084.
2. Fu-Tsun Wei, Green's funct ions on Mumford curves, Mathmat ische Annalen vol. 370 Issue 3-4 (2018) 1571-1605.
3. Chih-Yun Chuang & Ting-Fung Lee & Fu-Tsun Wei* & Jing Yu, Brandt matrices and theta series over global funct ion f ields, Memoirs of the American Mathemat ical SocietyVolume 237 Number 1117 (2015).

# 三、傑出博士論文獎

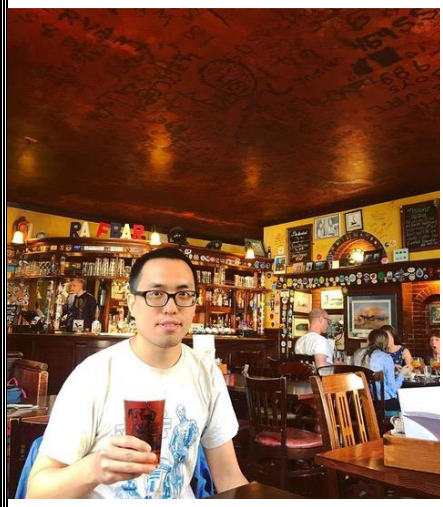
## 金牌獎

## 蘇瑋栢 博士

**論文：包極小拉格朗日子流形與拉格朗日均曲率流孤立子的穩定性**

Stability of Minimal Lagrangian Submanifolds and Soliton Solutions for Lagrangian Mean Curvature Flow

**論文指導教授：李瑩英教授。**



**學歷：**

國立臺灣大學數學系博士, 2014-2019
國立臺灣大學數學系碩士, 2012-2014
國立臺灣大學數學系學士, 2008-2012

**研究興趣：**

I am interested in the existence and uniqueness of special Lagrangian submanifolds and Lagrangian mean curvature flow in Calabi—Yau manifolds. The guiding principle in this direction is the Thomas—Yau conjecture (and the modified version by Joyce), which states that there exists a stability structure in the derived Fukaya category so that the stable objects are generated by special Lagrangian submanifolds. Therefore, my ultimate goal is to use the techniques from Geometric Analysis, Algebraic Geometry, and Category Theory to tackle Thomas—Yau conjecture.

## 論文工作介紹

Stability provides important information about critical points of some functionals. In this thesis, the class of functionals we are interested in are the f-volume functionals defined on the space of Lagrangian submanifolds in a K\"ahler manifold X, where f is a function on X. The critical points for the f-volume functional are called the f-minimal Lagrangian submanifolds, which are

generalizations of minimal Lagrangian submanifolds and soliton solutions for Lagrangian mean curvature flow. We study two different notions of stability with respect to the f-volume functional, namely the linear stability and dynamic stability.

The linear stability concerning the positivity of second variation of f-volume functional at an f-minimal Lagrangian submanifold. We derive a second variation formula for f-minimal Lagrangian submanifolds, which is a generalization of the second variation formula by Chen and Oh. Using this we obtain stability criterions for f-minimal Lagrangian submanifolds in gradient K\"ahler--Ricci solitons. In particular, we show that expanding and translating solitons for Lagrangian mean curvature flow are f-stable.

The dynamic stability on the other hand regarding the existence and convergence of the negative gradient flow of the f-volume functional, the generalized Lagrangian mean curvature flow, starting from an initial data nearby a critical point. Since the examples of f-minimal Lagrangians we are most interested in are complete noncompact, we first prove a short-time existence for asymptotically conical Lagrangian mean curvature flow. Then we give some long-time existence and convergence results for equivariant, almost-calibrated, asymptotically conical Lagrangian mean curvature flow in $C^{\square}$.

## 著作目錄

1. f-minimal Lagrangian Submanifolds in Kähler Manifolds with Real Holomorphy Potentials, 2019, available at arXiv:1901.00259, to appear in International Mathematics Research Notices.
2. Mean curvature flow of asymptotically conical Lagrangian submanifolds, 2019, to appear in Trans. of AMS.

## 推薦函節錄

兩位學者對蘇瑋栢博士論文的綜合評語：

1. In this thesis, Dr. Su Wei-Bo began the investigation with the derivation of the second variation of a f-volume functional at a f-minimal Lagrangian submanifold in a Kahler manifold. This lead Dr. Su to generalize previous results due to professors Chen and Oh. Upon this, Dr. Su went on to study the linear stability concerning the positivity of the second variation in this new content, as well as the dynamical stability regarding the existence and convergence of the negative gradient flow of the f-volume flow.

   Even at this fundamental level, it is apparent that Dr. Su has mastered basic notions in Complex and sympletic geometries. This alone demonstrates the width of knowledge which is import ant for Dr. Su's future pursuit in research.

   Section 3 of the thesis establishes the second variation formula with respect to the f-volume functional, thus leading to the respective linear stability theory for the subject. This piece of research which generalizes previous analog results of Chen and Oh is very down to earth, with all details being written down very concisely.

   In the study of the dynamic stability, the respective f-minimal Lagrangians which Dr. Su considered are non-compact, which give rise to non-trivial difficulty. To overcome this, Dr. Su was able to locate the right piece s of tools in classical geometric analysis (due to Grigor'yan, Saloff-Coste) to resolve the difficulty which arises from the open end of an asymptotically conical Lagrangian submaifold. This reflects that Dr. Su could successfully combine the techniques of difficult geometric analysis (such as Heat kernel estimate in Theorem 1.2.1 and weighted Schauder estimate etc) with those in Sympletic geometry.

   All and all, this thesis clearly demonstrates the width and depth of Dr. Su knowledge in areas of sympletic geometry and geometric analysis. Moreover, the results which Dr Su obtained are of interest to important

scholars such as professor Joyce.

2. This is an interesting and solid PhD thesis. The author basically studies the stability of Lagrangian mean curvature flow (LMCF). The first part of the thesis deals with the so called linear stability about f-minimal Lagrangian submanifolds ("f-minimal" is a notion of "minimal" with respect to a weighted volume functional $V_f$). The author deduces the second variation formula for $V_f$ and discusses the associated stability. The second part of the thesis discusses the so called dynamical stability (i.e., if we start with a certain type of Lagrangian submanifold L near a model example L_0, then L will converge to L_0 under the LMCF). The author considers the class of asymptotically conical (AC in short) Lagrangian submanifolds under the flow called AC LMCF. He shows the short time existence and discusses the dynamical stability in some cases. Technically he needs to deal with the long time existence and prove the convergence through some blow-up analysis. LMCF is a special kind of mean curvature flow of high codimension. It is hard to get general results on the long time existence and the convergence. So it is already not easy for the author to prove such a stability result in some restricted class of AC Lagrangian submanifolds.

綜觀兩位學者對蘇瑋栢博士論文所給高度肯定之評語，本會特頒傑出博士論文獎金牌獎，以茲表揚。

# 四、傑出博士論文獎

## 銀牌獎

## 郭驥 博士

**論文：整函數的整除問題** Quotient Problems for Entire Functions

**論文指導教授：王姿月教授。**

**學歷：**

國立清華大學數學系博士, 2015-2019
國立臺灣大學環工所碩士, 2012-2015
北京師範大學數學系學士, 2007-2011

**研究興趣：**

我的研究興趣主要在 diophantine geometry 以及 Nevanlinna theory，目前我主要研究的問題有 linear recurrences 和對應複變函數的 greatest common divisor problem、quotient problem 以及 q-th root problem。

## 論文工作介紹

Let {F(n)} and {G(n)} be linear recurrence sequences. It is a well-known diophantine problem to decide the finiteness of the set N of natural numbers such that their ratio F(n)/G(n) is an integer. We want to study an analogue of such a quotient problem in the complex situation.

First, let f and g be entire functions which are multiplicatively independent. We want to determine whether fn-1 is divisible by gn-1 for infinitely many n. This is an application of the GCD estimate of fn-1 and gn-1, i.e. the Nevanlinna counting function for the common zeros of these two sequences of functions. For this estimate, we need to formulate a truncated Nevanlinna second main theorem for effective divisors and explicitly compute the constants involved for a blow-up of P1*P1 along a point. Finally, we generalize the quotient problem to a multi-variable version as well as moving target version.

## 著作目錄

1. J. Guo and J. T.-Y. Wang, Asymptotic gcd and divisible sequences for entire functions, Transactions of the American Mathematical Society, 371(2019), no. 9, 6241-6256.
2. J. Guo, The quotient problem for entire functions, Canadian Mathematical Bulletin, to appear.
3. J. Guo, The quotient problem for entire functions with moving targets, preprint.

## 推薦函節錄

兩位學者對郭驥博士論文的綜合評語：

1. 被推薦人的博士論文研究主題是屬於定義在函數域(function fields)上的丟翻圖問題(Diophantine problems)。『丟翻圖問題』的研究一直是數論領域的中心課題，很多著名的數論問題如費馬最後定理(Fermat's Last Theorem)等都屬於此範疇。租略地說，古典的丟翻圖問題可理解為代數方程維求解的問題，一個重要的現象是函數域中有許多相似於數域(整數、非整數等)的概念，因此，研究定義於函數域上的丟翻圖問題在數論以及(複變)函數理論是一項有意思且重要的課題。

   通過 C. Osgood 以及 P. Vojta 等數學家的工作，我們了解到複變函數域裡的 Nevanlinna 值分佈理論與丟翻圖逼近論有著驚人的平行對照結果。在數論問題尤其是丟翻圖問題的研究上， 丟翻圖逼近論是不可或缺的工具，因此，研究定義於函數域上的丟翻圖問題， Nevanlinna 理論扮演著重要的角色。很自然地，Nevanlinna 理論也是郭博士的論文裡面主要的工具。

   郭博士的阱究成果主要可分兩個方面來談：

   第一項結果是關於複變函數域裡兩個全純函數 (entire functions) 序列的元素之間公因子(GCD) 的問題。更精維地說，給定全純函數 $f$ 和 $g$，這裡所要研究的問題是當 $n$ 在正整數集合中變化時，求 $f^n-1$ 以及 $g^n-1$ 共同零根數 $N(f^n-1,g^n-1,r)$ (Nevanlinna GCD) 的上界。在數域的情況，已有許多文獻探討相對應的問題，然而，依現有的數論理論似乎必須假定數論中一個深刻且困難的猜想-- Vojta 猜想才能獲得這個問題的最終解答。

一般而言，除非 $f$ 和 $g$ 滿足一些明顯的代數關係，否則 $N(f^n-1,g^n-1,r)$ 相較於 $f^n$ 與 $g^n$ 的 Nevanlinna 高度函數 $T_{\{f^n\}}(r)$ 以及 $T_{\{g^n\}}(r)$ 應該很小，其詳細的意義請參考其論文。和數域的情況相同，我們仍然可以假設 Vojta 猜想來證明函數域中這項命題成立。在郭博士的論文裡， 他利用 Ru-Vojta 最近的結果，成功地繞過 Vojta 猜想，從而得到 $N(f^n-1,g^n-1,r)$ 的一個上界。雖然比預期的上界弱，卻是不假設 Vojta 猜想之下到目前所能得到的最好結果。這裡得到的上界已足夠讓他解決在函數域中對應古典 Pisot 猜想的一個類比問題，除了上述 $N(f^n-1,g^n-1,r)$ 的上界之外，這是郭博士論文另一項突出的地方。

郭博士的第二項研究結果是應用上述關於 $N(f^n-1,g^n-1,r)$ 的上界進一步探討函數域兩個線性遞迴(linear recurrence)全純函數序列其元素之間的商是否仍為全純函數的問題。在這裡郭博士所處理的線性遞迴序列的係數不見得是常數，他需要考慮函數係數為 “慢增長” (slow growth)的情況，因此技術上面臨新的困難，而郭博士克服了困難，成功地獲得預期的結果，這也是其博士論文一項值得肯定的成果。

郭博士的工作對於函數域中 Pisot 猜想的問題成功地給出解答，尤其應用 Ru-Vojta 的結果得到 $N(f^n-1,g^n-1,r)$ 有效的上界則是一項重要的突破。由所附的著作目錄可知郭博士的論文內容已在數學國際期刊發表了三篇論文，其中一篇發表於 Transactions of AMS 是美國數學學會相當優良且重要的數學期刊。總結來看，這是相當好的博士論文。

2. As noticed by Osgood and Vojta, there is a striking analogy between statements in Diophantine geometry and those in the Nevanlinna theory. In Dr. Guo’s Ph.D. thesis, he considered problems about quotients of entire functions defined in a similar way as linear recurrence sequences and obtained results for these quotients that are analogous to the results for linear recurrence sequences. Note that the problems considered in the thesis are not new. For example, in GCD bounds for analytic functions published on IMRN in 2017, Dr. Guo’s thesis advisor has already obtained an average version of Theorem 1.11 in the thesis. However, those existing results are not strong enough to yield the main results in the thesis. Instead, Dr. Guo needed to improve several results from the Nevanlinna theory in order to establish his theorems.

Schimidt's subspace theorem for varieties is also needed.

There are four main theorems in the thesis, namely, Theorems 1.9, 1.11, 1.12, and 1.13. Among them, Theorem 1.9 and Theorem 1.11 were published, jointly with Dr. Guo's thesis advisor, on the Transactions of the American Mathematical Society, which is one of the most respected journals. (As a matter of fact, Theorem 1.9 can be regarded as a corollary of Theorem 1.11 as the GCD estimate from Theorem 1.11 plays a crucial role in the proof of Theorem 1.9). Theorem 1.12 was published on the Canadian Mathematical Bulletin (CMB) very recently. Theorem 1.13 has been submitted for publication. However, because Theorem 1.13 is more or less a minor improvement of Theorem 1.12, under a normal circumstance, it is doubtful that Theorem 1.13 can be published on a journal with much higher reputation than the CMB. (From the publication date of the paper on the CMB and the completion date of the Ph.D. thesis, it appeared that after submitting Theorem 1.12 for publication, Dr. Guo realized that his method can be extended to yield a more general result.)

Overall, I think the thesis is of solid quality. The results are quite interesting and significant. In addition, the thesis was very well-prepared as I can hardly find any typos or missed punctuation marks.

綜觀兩位學者對郭驥博士論文所給高度肯定之評語，本會特頒傑出碩士論文獎金牌獎，以茲表揚。

# 五、傑出碩士論文獎

## 銀牌獎

## 李冠輝 碩士

**論文：唐納森流和平均曲率流在四維超凱勒流型的穩定性**

The Stability of Donaldson's Flow and Mean curvature Flow in Hyperkähler Four Manifolds

**論文指導教授：崔茂培教授。**

**學歷：**

國立臺灣大學數學所碩士, 2017-2019

國立臺灣大學數學系學士, 2013-2017

**研究興趣：**

主要研究領域是微分幾何，其中包含了Ricci Flow Mean Curvature Flow以及其他各種的geometic evolution equations。

在研究hyperkahler four manifold上的Donaldson's flow中，亦接觸了Kahler幾何Holonomy Theory等理論。

## 論文工作介紹

In recent year, Wang and Tsai proposed a series of paper about the stability of mean curvature flow about strongly stable submanifolds. They show that if a submanifold is close to the strongly stable submanifold then the mean curvature flow exists for all time and converges smoothly. On the other hand, Donaldson used moment map and diffeomorphism to construct lots of geometric evolution flows. In particular, the hyperkähler four manifold case was explicitly discussed by Song and Weinkove. They found that Donaldson's flow is similar to the mean curvature flow in this case. In this thesis, we discuss the Donaldson's flow in detail and prove a result similar to the Wang and Tsai's result.

## 推薦函節錄

兩位學者對李冠輝碩士論文的綜合評語：

1. This is an excellent Master thesis. Its quality is much better than a usual Master thesis. The author had to be familiar with a lot of background material such as complex and hyperkahler structures in geometric side and curvature flows in analytic side before he could work on some interesting problems. The study of hyperkahler 4-manifolds has attracted much attention in recent years. The author ends up to be able to prove a stability result about the special hyperkahler flow (Donaldson's Flow) of a certain class of Lagrangian surfaces on a hyperkahler 4-manifold. The reviewer believes that the result is new and can be published in an international decent journal. The proof involves many estimates to build up long time existence and the convergence of the flow. This shows the author's prowess in analytic techniques. In fact it looks like a PhD thesis.

2. In this thesis, Dr. Lee mainly gave a detailed exposition of the constructions of geometric evolution flows due to Do naldson, Son and Weinkove. Since the work of professors Song and Weinkove emphasized the part icular case of hyperkahler manifolds. As is well-known, the subject of hyperkahler manifolds is a difficult one. This difficulty is partially due to the fact that the existence of a hyperkahler metric is based on the classical Calabi-Yau's theorem in geometric analysis, and thus is not explicitly expressed. In any case, Dr. Lee succeeded in giving a very careful and clear exposition of the fundamental material, as well as the more up-to-dated current researches due to important scholars at the international level. I would like to stress that the basic subject about hyperKahler manifold alone is already a challenge for a Master-student at the common level. So, it is apparent from the content of this thesis that Dr. Lee has mastered all the technical aspects as involved. Moreover, it is worthwhile to mention that Dr. Lee was able to push the constructions one step forward. This shows that Dr. Lee is ready to step on the road of serious research in the future.

綜觀兩位學者對李冠輝碩士論文所給高度肯定之評語，本會特頒傑出碩士論文獎銀牌獎，以茲表揚。

# 六、傑出碩士論文獎

## 銀牌獎

## 阮赫端 碩士

**論文：譜的幾何性質和ρ-拉普拉斯的梯度設計**

Spectrum Geometric Properties And Gradient Estimates For P-Laplacian

**論文指導教授：宋瓊珠教授。**

**學歷：**

國立清華大學數學所碩士 2017-2019

河內第二師範大學數學系學士 2012-2016

**研究興趣：**

Geometric Analysis and Partial Differential Equations

## 論文工作介紹

In this thesis, we study the properties of the spectrum geometric and gradient estimates for p-Laplacian on complete Riemannian manifolds with the Ricci curvature is bounded from below. First, we prove a splitting type theorem for the Laplacian. Then, we give a local gradient estimate for the positive p-eigenfunction associated to the first eigenvalue of the p-Laplacian. Moreover, we show global sharp gradient estimates for p-eigenfunctions. On the other hand, we first derive a Li-Yau type gradient estimate for the positive solutions to the p-Laplacian heat equation. Finally, we prove that the first eigenvalue of the p-Laplacian is strictly monotone increasing and differentiable almost everywhere along with the Ricci-Bourguignon flow under some different curvature assumptions.

## 著作目錄

1. H. T. Dung, Gradient estimates and Harnack inequalities for Yamabe-type

parabolic equations on Riemannian manifolds, Differential Geometry and its Applications (60), 2018, 39-48. (SCIE)

2. H. T. Dung, N. T. Dung, Sharp gradient estimates for a heat equation in Riemannian manifolds, to appear Proceedings of the American Mathematical Society, 2019. (SCI)
3. H. T. Dung, Monotonicity of eigenvalues of the *p*-Laplace operator under the Ricci-Bourguignon flow (accepted), Kodai Mathematical Journal, 2019. (SCIE)

## 推薦函節錄

兩位學者對阮赫端碩士論文的綜合評語：

1. This is a very solid Master thesis. The author studies the properties of the spectrum and the gradient estimate for the p-Laplacian on a complete Riemannian manifold with Ricci curvature bounded from below. He obtains many results as follows. He first shows a splitting type theorem for the Laplacian. Applying known techniques for the Laplacian, he gives a local gradient estimate for the positive p-eigenfunction associated to the first eigenvalue of the p–Laplacian and global sharp gradient estimates for p-eigenfunctions. He also derives a Li-Yau type gradient estimate for the positive solutions to the p-Laplacian heat equation and the first eigenvalue of the p-Laplacian is strictly monotone increasing and …. This thesis is much longer than a usual Master thesis, collecting many small results together. The p-Laplacian for p>2 is mildly interesting in my opinion.

2. In the thesis entitled "Spectrum Geometric Properties and Gradient Estimates for p-Laplacian, Mr. Dung studied the current wel 1-known results which are generalizations of those classical ones in Geometric Analysis as developed in the 1980's. Concrete results which are reported by Mr. Dung here include, in particular, gradient estimate for the p-laplacian (due to Dung Oat, Wang-Zhang, Li-Wang, Sung-Wang etc.), as well as that for the p-Laplacian heat equations. All in all, it is apparent to me that: in order to achieve this, Mr. Dung had first, as a necessary preparation, to master the majority of the basic material in the extremely well-known famous book on Geometric Analysis by professors Yau and Schoen (as well as that by professor Peter Li). I would like to stress that the

duration of Master-level study is relatively short, and it is unusual that Mr. Dung could acquire such an overwhelming amount of deep and difficult techniques in classical geometric analysis in his Master-level study. Not only that, Mr. Dung also tried, in section 5. 2 of his thesis, to slightly improve those well-known gradient estimates for heat equation in the manifold setting. This is a brave action by Mr. Dung, and this counts as Mr. Dung very first step towards his future serious research in mathematics.

It is my opinion that the level of this Master-level thesis by Mr. Dung clearly shows that Mr. Dung is ready to pursuit research at a genuine level in the near future. As I see it (from the evidences as seen in this thesis), Mr. Dung has acquired the basic training of Geometric Analysis even at the beginning Ph.D. level, though he is now just a graduate at the Master-degree level. It is my genuine hope that Mr. Dung will have independent and original contribution to mathematical research in his near future.

I am very impressed by the breath of training and expertise knowledge of Mr. Dung.

綜觀兩位學者對阮赫端碩士論文所給高度肯定之評語，本會特頒傑出碩士論文獎銀牌獎，以茲表揚。