The Mathematics of Shuffling Cards

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Time: 3:30PM - 4:30PM, March 8 (Wed.)

Lecture 1: Adding Numbers and Shuffling Cards

Abstract: When numbers are added in the usual way, 'carries' accrue along the way. Carries make a mess and it is natural to ask ‘how often is there a carry and if there is just a carry is it more or less likely that there will be carry next?’ It turns out that carries form a Markov chain with 'an amazing transition matrix.' This same matrix comes up in analyzing the usual method of riffle shuffling cards. I will explain the connection and 'the seven shuffles theorem' 'in English' for a general audience.

Time: 3:30PM - 4:30PM, March 9 (Thur.)

Lecture 2: Adding a List of Numbers (and Other Determinantal Point Processes)

Abstract: There is interesting math to be found in tracking the carries when a single column of digits is added(!) It turns out that the carries process forms a determinantal point process. Such processes occur in describing the energy spectrum of atoms, in random matrix theory, in describing the zeros of Riemann's zeta function and in hundreds of other applied problems. They admit a general theory. Going back to carries, it turns out that 'carries are cocycles' and the whole story works for any finite group and subgroup (and more generally yet). As I will explain, 'it's all related to shuffling cards.'

Time: 3:30PM - 4:30PM, March 10 (Fri.)

Lecture 3: Shuffling Cards and the Geometry of Hyperplane Arrangements

Abstract: Consider a collection of affine hyperplanes in d-dimensional Euclidean space. This divides space in chambers (points not on any of the hyperplanes) and faces. There is a natural projection operation (Tits projection): given a face and a chamber, there is a unique chamber, adjacent to the face and closest to the starting chamber. Picking faces randomly induces a Markov chain on chambers. Believe it or not, this encompasses a huge collection of natural Markov chains--including riffle shuffling, but also the Tsetlin library of computer science, the Ehrenfests urn process of statistical physics and hundreds of others. There is a complete theory of such chains (closed form expressions for all the eigenvalues, sharp rates of convergence, ...). Generalization of this set up (due to Aguiar-Mahadahn) lead to abstractions of general ideas of modern algebra (such as Hopf algebras and Lie algebras). Again, I will try to explain things for a general mathematical audience.